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Heat transfer in flat-plate boundary layers: a correlation for laminar, transitional, and turbulent flow

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Heat transfer in flat-plate boundary layers: a correlation for laminar, transitional, and turbulent flow

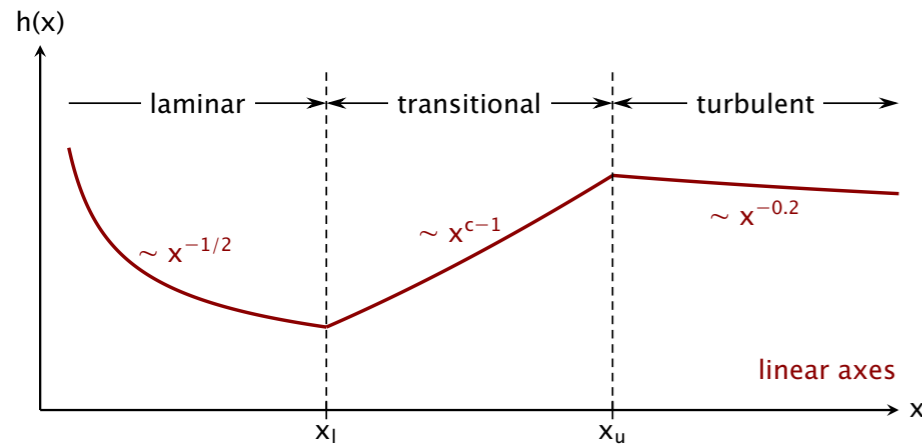
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We Can't Neglect the Transition Region

Transition region and laminar region have similar length



The two step, laminar-then-turbulent model is incorrect!

Summary of the Correlation

For smooth, sharp-edged, flat plates with zero pressure gradient and either uniform wall temperature (UWT) or uniform heat flux (UHF)

Combining formula

$$Nu_x(Re_x, Pr) = \left[Nu_{lam}^5 + \left(Nu_{trans}^{-10} + Nu_{turb}^{-10} \right)^{-1/2} \right]^{1/5} \quad \text{Eq. (9)}$$

Laminar region

$$Nu_{lam}(Re_x, Pr) = \begin{cases} 0.332 Re_x^{1/2} Pr^{1/3} & \text{UWT} \\ 0.453 Re_x^{1/2} Pr^{1/3} & \text{UHF} \end{cases}$$

With an unheated starting length of x_0 (UHF or UWT), use

$$Nu_{lam}(Re_x, Pr) \cdot \left[1 - (x_0/x)^{3/4} \right]^{-1/3}$$

Transition region

$$Nu_{trans}(Re_x, Pr) = Nu_{lam}(Re_l, Pr) \cdot (Re_x/Re_l)^c$$

Re_l is the Reynolds number at onset of transition, x_l

$$c = 0.9922 \log_{10} Re_l - 3.013 \quad \text{for } Re_l < 5 \times 10^5$$

Turbulent region (UHF and UWT)

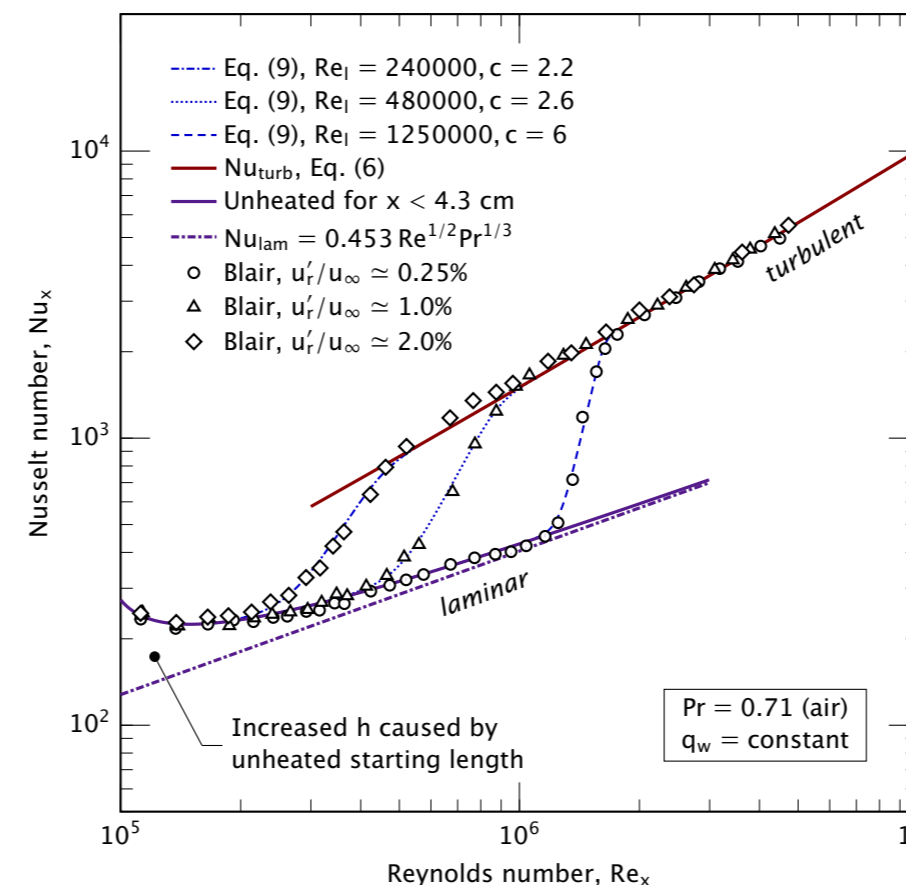
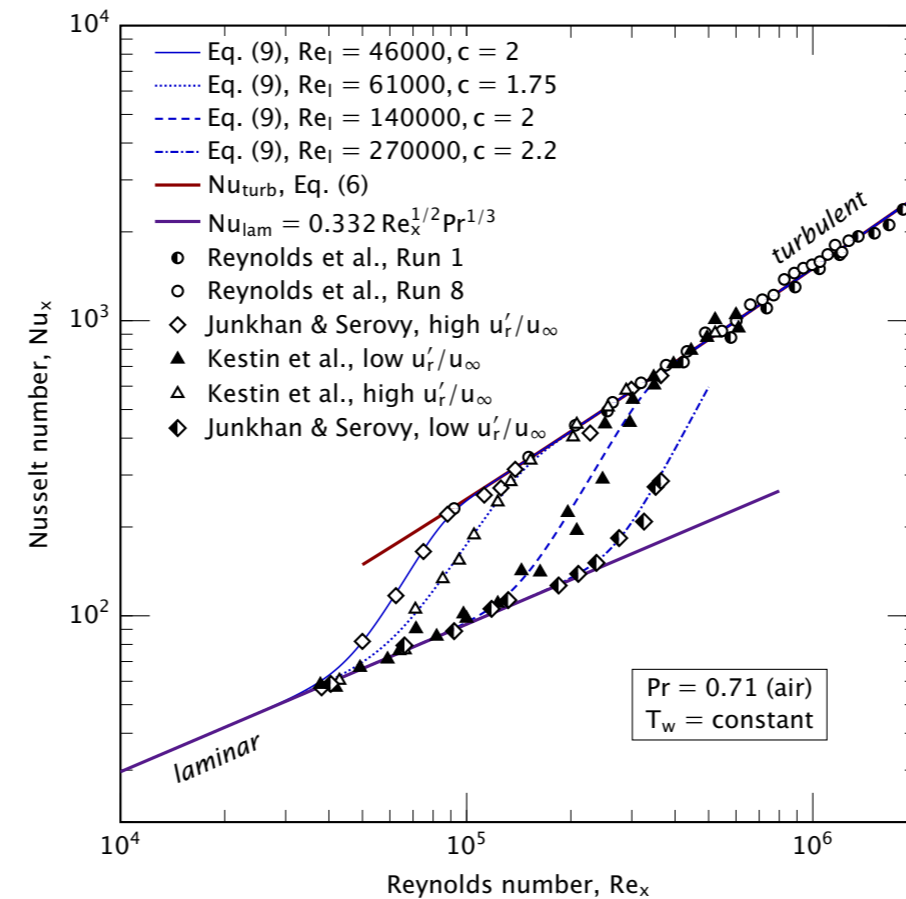
$$Nu_{turb}(Re_x, Pr) = \frac{Re_x Pr (C_f/2)}{1 + 12.7 (Pr^{2/3} - 1) \sqrt{C_f/2}} \quad \text{Eq. (6)}$$

$$C_f(Re_x) = \frac{0.455}{[\ln(0.06 Re_x)]^2}$$

For gases only, the following equation has similar accuracy

$$Nu_{turb}(Re_x, Pr) = 0.0296 Re_x^{0.8} Pr^{0.6} \quad \text{for gases}$$

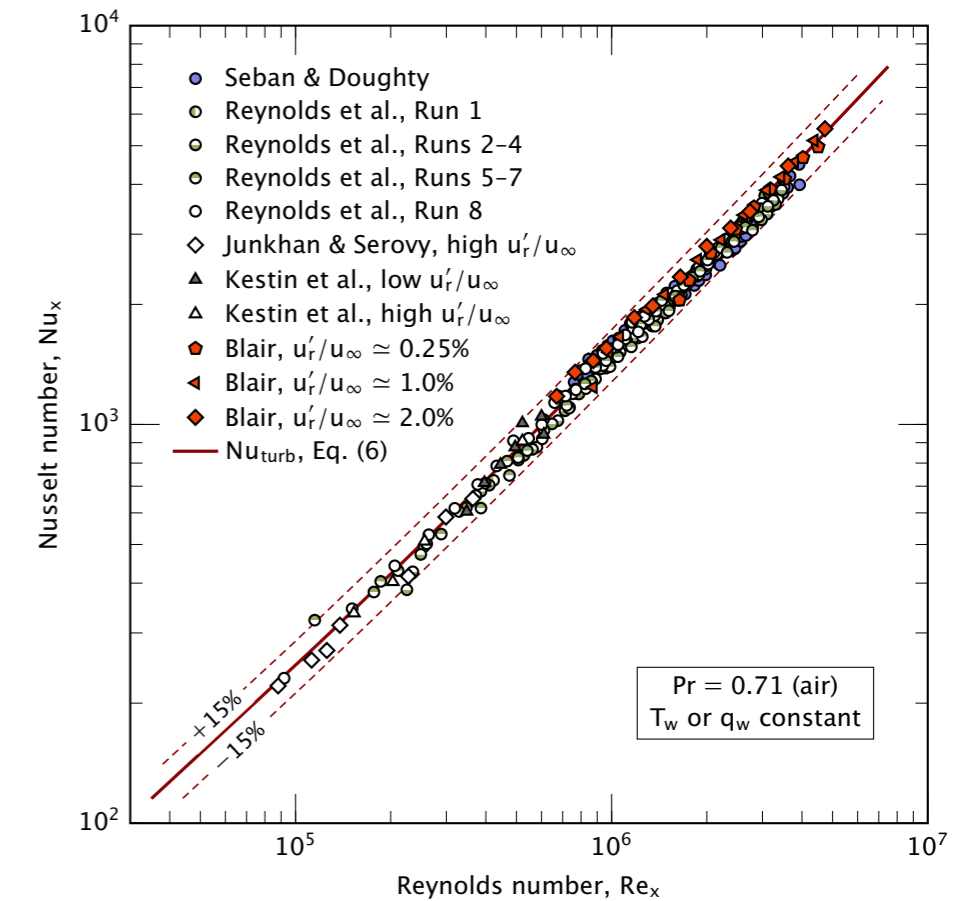
Typical Results (more data & fluids in paper)



Data from Multiple Independent Experiments

$0.7 \leq Pr \leq 257$ $4,000 \leq Re_x \leq 4,300,000$
free-stream turbulence levels up to 5%

Fully turbulent air data fit to std. dev. of $\pm 5.5\%$



Classical Colburn analogy (1933)

Not recommended: Colburn's $St = (C_f/2) Pr^{-2/3}$ was based on b.l. data for air and does not support a wide range of Pr. Colburn's suggestion to use it for laminar flow compared a UWT formula to misplotted UHF data.

Similarity solution for UHF laminar b.l.

This result (Fage & Falkner, 1931; Imai, 1958) is not widely known

$$Nu_{UHF} = 0.4587 Re_x^{1/2} Pr^{1/3} \quad \text{similarity solution}$$

but close to integral-method (replace 0.4587 by 0.4535). Pre-1950, wall boundary conditions often overlooked (Colburn 1933; Jakob & Dow 1946)