Heat transfer in flat-plate boundary layers: a correlation for laminar, transitional, and turbulent flow

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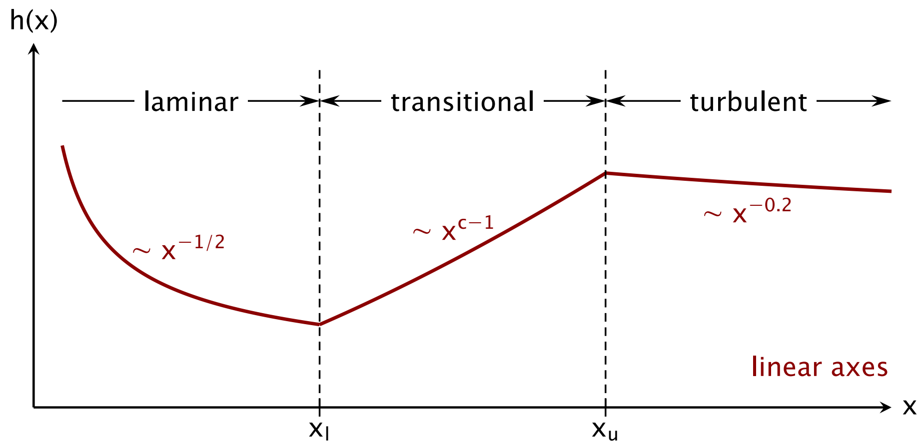
We Can't Neglect the Transition Region

Transition region and laminar region have similar length



The two step, laminar-then-turbulent model is incorrect!

Summary of the Correlation

For smooth, sharp-edged, flat plates with zero pressure gradient and either uniform wall temperature (UWT) or uniform heat flux (UHF)

Combining formula

$$\mathrm{Nu}_x(\mathrm{Re}_x,\mathrm{Pr}) = \left[\mathrm{Nu}_{lam}^5 + \left(\mathrm{Nu}_{trans}^{-10} + \mathrm{Nu}_{turb}^{-10}\right)^{-1/2}\right]^{1/5} \qquad \text{Eq. (9)}$$

Laminar region

$$\mathrm{Nu}_{lam}(\mathrm{Re}_x,\mathrm{Pr}) = \begin{cases} 0.332\,\mathrm{Re}_x^{1/2}\mathrm{Pr}^{1/3} & \text{UWT} \\ 0.453\,\mathrm{Re}_x^{1/2}\mathrm{Pr}^{1/3} & \text{UHF} \end{cases}$$

With an unheated starting length of x_0 (UHF or UWT), use

$$\mathrm{Nu}_{lam}(\mathrm{Re}_x,\mathrm{Pr}) \cdot \left[1 - (x_0/x)^{3/4}\right]^{-1/3}$$

Transition region

$$\mathrm{Nu}_{trans}(\mathrm{Re}_x,\mathrm{Pr}) = \mathrm{Nu}_{lam}(\mathrm{Re}_l,\mathrm{Pr}) \cdot (\mathrm{Re}_x/\mathrm{Re}_l)^c$$

Re_l is the Reynolds number at onset of transition, x_l

$c = 0.9922 \log_{10} \mathrm{Re}_l - 3.013$ for $\mathrm{Re}_l < 5 \times 10^5$

Turbulent region (UHF and UWT)

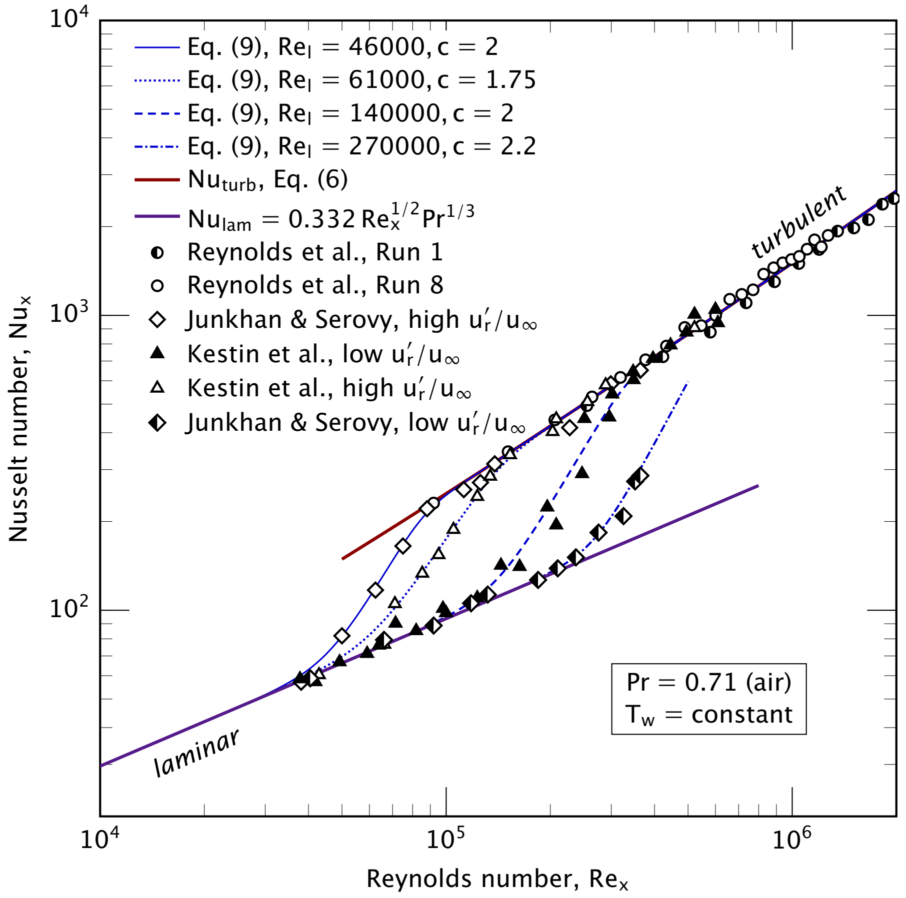
$$\mathrm{Nu}_{turb}(\mathrm{Re}_x,\mathrm{Pr}) = \frac{\mathrm{Re}_x \mathrm{Pr}\,(C_f/2)}{1 + 12.7(\mathrm{Pr}^{2/3} - 1)\sqrt{C_f/2}} \qquad \text{Eq. (6)}$$

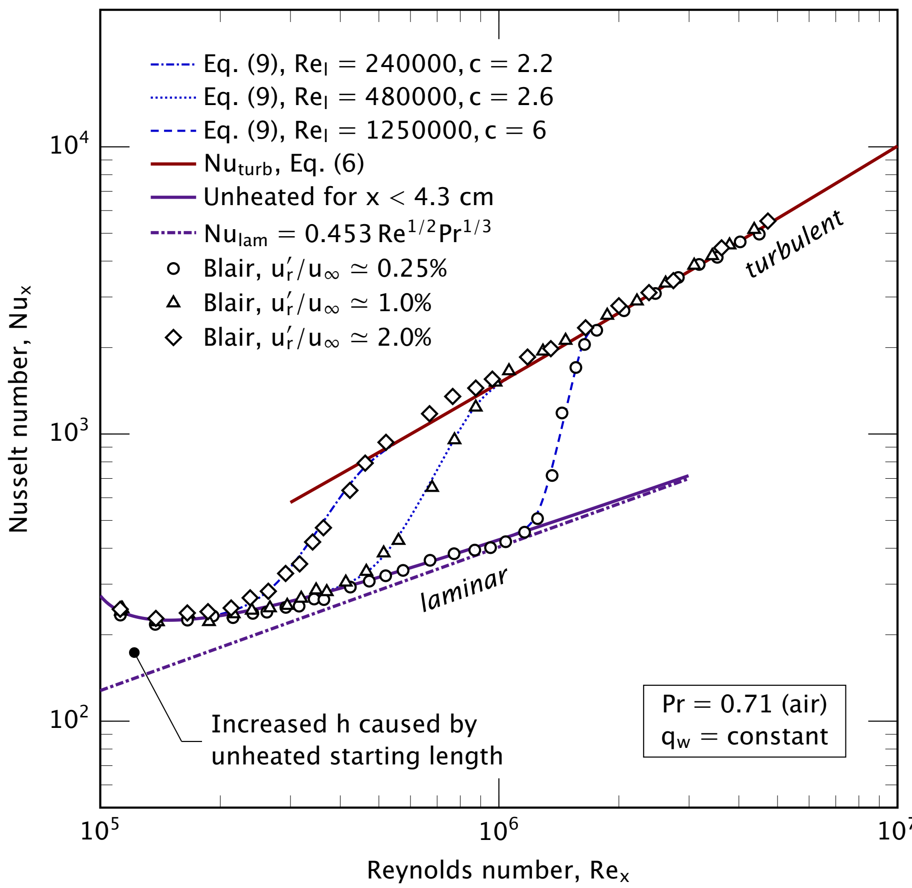
$$C_f(\mathrm{Re}_x) = \frac{0.455}{\left[\ln(0.06\,\mathrm{Re}_x)\right]^2}$$

For gases only, the following equation has similar accuracy

$$\mathrm{Nu}_{turb}(\mathrm{Re}_x,\mathrm{Pr}) = 0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{0.6} \text{ for gases}$$

Typical Results (more data & fluids in paper)



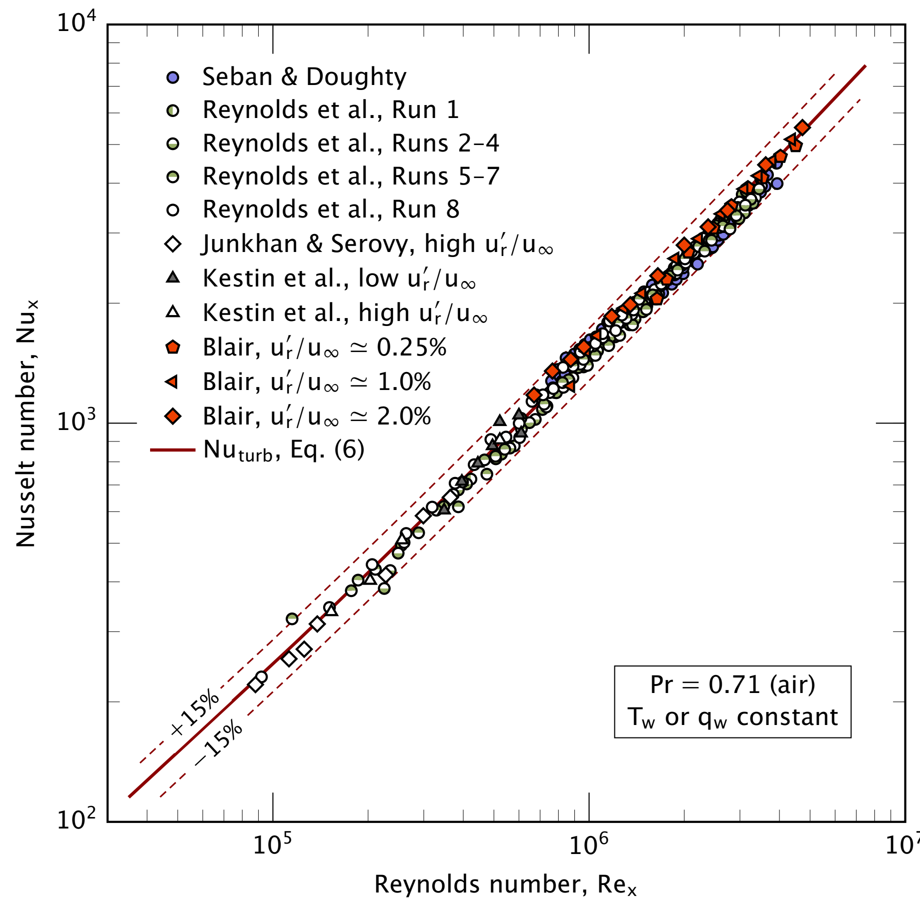


Data from Multiple Independent Experiments

$0.7 \leqslant \mathrm{Pr} \leqslant 257$ $\quad 4,000 \leqslant \mathrm{Re}_x \leqslant 4,300,000$

free-stream turbulence levels up to 5%

Fully turbulent air data fit to std. dev. of ±5.5%



Classical Colburn analogy (1933)

Not recommended: Colburn's $\mathrm{St} = (C_f/2)\,\mathrm{Pr}^{-2/3}$ was based on b.l. data for air and does not support a wide range of Pr. Colburn's suggestion to use it for laminar flow compared a UWT formula to misplotted UHF data.

Similarity solution for UHF laminar b.l.

This result (Fage & Falkner, 1931; Imai, 1958) is not widely known

$$\mathbf{Nu_{UHF} = 0.4587\,Re_x^{1/2}Pr^{1/3}} \quad \text{similarity solution}$$

but close to integral-method (replace 0.4587 by 0.4535). Pre-1950, wall boundary conditions often overlooked (Colburn 1933; Jakob & Dow 1946)