The Effects of Spending Rules and Asset Allocation on Non-Profit Endowments[1]

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1. We quantify the long-run impact of interactions between an endowment's spending policy and its asset allocation decisions on the endowment's risk/reward profile.
2. We compare and contrast the spending and asset allocation policies of four university endowments and show how certain spending and asset-allocation policy combinations can lead to greater gains at the cost of larger worst-case losses.
3. We explore the specific relationship between investing and spending for each endowment, both analytically and empirically via Monte Carlo simulations, and find that no spending rule, investment risk profile, or university utility preference is objectively superior.

Abstract:

The long-run impact and implications of an endowment's spending policy and asset allocation decisions are examined. Using a dynamic model, the authors explore how different endowment spending rules influence the dynamics of an endowment's size and future spending. They find that different parameters within each spending rule have significant long-term impact on wealth accumulation and spending capacity. Using Merton's (1993) endowment model and compiled asset allocation data, they estimate the intertemporal preferences and risk aversion of several major endowments and find significant variation across endowments in their propensity to increase portfolio risk in response to increased spending needs.

Keywords: Non-profit organizations; endowment spending; asset allocation

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INTRODUCTION

As of 2020, nonprofit endowments in the U.S. manage over $800 billion, with an annual growth in cumulative assets under management (AUM) of around 3%.[1] While there has been extensive work on the investment risk and return of endowment funds as well as their optimal investment policies, there has been relatively little work on the spending policies of these funds or their effect on endowments.[2] For the vast majority of endowment funds, spending policies and practices are key not only to the sustainable support of the parent organization's operations, but to long-term fund growth and the viability of the organization. The goal of this paper is to evaluate the impact of spending policies and asset allocation on endowment growth and risk preference.

We begin by documenting five major spending policies most frequently used by endowment funds: three variations of the Tobin (1972) spending rule, a "flat" spending rule, and a bounded adjustment rule. Using a dynamic model, we simulate the annual endowment investment returns, spending amount, and assets under management (AUM) under all five spending policies. In our simulations, we use two sets of endowment asset classes: the Stocks, Bonds, Bills, and Inflation (SBBI) dataset and a compiled list of commonly invested indexes in equity markets, fixed income, hedge funds, private equity, real estate, and real assets.

To study the quantitative predictions of our model, we use representative asset allocations of major endowments—Harvard, Yale, Stanford, and MIT—as well as several corner case portfolios. We present a variety of evaluation metrics, such as benchmark and loss measures, to further analyze each spending policy. We find that different spending policies lead to large differences in endowment values over time as well as statistically significant differentials in spending percentage curves. This result highlights the importance of spending policies, and suggests that governing bodies of nonprofit organizations should carefully consider policy implementation.

We then assess the quantitative importance of the key attributes of each spending policy. Spending policies differ along three principal dimensions. First, most rules peg spending in a certain proportion to the amount spent in prior years, in addition to the market value of the endowment. All else equal, a higher weight on prior spending and a lower weight on current market value lead to a lower volatility of spending over time. Second, spending rules differ in their exact treatment of adjustment for inflation. Third, spending rules typically adopt a long-term spending rate as a fraction of their endowment value. For example, a 5% flat spending rule would prescribe spending 5% of the endowment's AUM each year. Our results show that the weighting split between prior spending and endowment value has the largest impact on future value of the endowment and its spending dynamics,

[1] Authors' estimates based on data in Form 990 filed by nonprofit endowment funds with the IRS for fiscal year 2020.

[2] For the most recent evidence on investment risk, returns, and asset allocations of nonprofit endowments, see Lo, Matveyev, and Zeume (2021) and Dahiya and Yermack (2021).

followed by the long-term spending rate and the inflation treatment, both of which nevertheless still produce sizable effects.

Endowments are inherently loss-averse, in order to protect the financial wellbeing of the parent organizations they serve. We thus employ several metrics to measure the worst outcomes of spending rules and asset allocation decisions. In particular, we use the maximum and average largest loss, as well as the maximum drawdown and its duration. We find that asset allocation, rather than a particular spending policy, has the most pronounced impact on the worst outcomes. Spending policy has the largest impact on the maximum largest loss and maximum drawdown.

Finally, we study the interdependence between endowment investing and spending. During market downturns or disaster events, such as the global COVID-19 pandemic, endowments must be prepared to allocate additional funds to cover a shortfall in other sources of revenue. Endowment investment performance may also suffer in the years affected by these downturns. It is therefore important to analyze the adjustments to spending based on changes in investment return, and to scrutinize the modifications to asset allocation and investment strategy based on the required changes in spending.

For each endowment, we use the Merton (1993) endowment portfolio choice model to estimate a frontier of risk aversion and marginal propensity to increase endowment wealth when spending needs change. We find that for comparable levels of risk aversion, Harvard and Stanford have a higher marginal utility of wealth with respect to their spending needs compared to Yale and MIT. This implies that if spending needs increase, or if there is a shortfall in the other sources of revenue, Harvard and Stanford are more likely to change their portfolio asset allocation in response to changing spending needs.[3]

Our analysis also shows that Harvard exhibits the highest level of risk aversion of the four major endowments considered. All else equal, this implies a lower marginal utility of wealth with respect to spending needs compared to other endowments. This finding is consistent with the recent episode when Harvard, due to its revenue loss during the COVID-19 pandemic, reduced its spending on capital projects and acquisitions from $903 million in FY 2019 to $627 million in FY 2020, without altering the risk profile of its endowment.[4] Finally, we also find that, out of the four endowments, MIT is least likely to change its endowment asset allocation and portfolio risk in response to changing spending needs.

LITERATURE REVIEW

Our paper is related to several strands of literature on endowment funds. First, we contribute to the literature on the interplay between investment decisions and sustainable spending, which dates back to Tobin (1974). The classic Merton (1993) model highlights the

[3] In 2020, Stanford adopted a slightly riskier investment portfolio that produced better than expected returns in order to alleviate the 19% decline in their FY 2020 income (see the 2020 Stanford University Investment Report). Harvard has also attributed its recently reported high investment returns to the greater risk of its investment strategy (see Harvard Management Company, Message from the CEO, October 2021.)

[4] See the Financial Report of Harvard University for fiscal year 2019-2020.

use of an endowment fund to hedge the risk of a university's income volatility. Gilbert and Hrdlicka (2013) argue that high endowment spending shifts risks to future generations. Dybvig and Qin (2019) focus on the preservation of endowment capital and derive sustainable spending policies that meet this objective. Dimmock, Wang, and Yang (2019) study the role of illiquid alternative assets in Merton's (1993) framework. More recently, Campbell and Sigalov (2021) adapt Merton's (1993) model to show that sustainable spending requirements put pressure on endowments to allocate capital to riskier assets in a low interest rate environment.[5]

Second, our work is related to the extensive literature on returns and asset allocations of endowment funds. Lerner, Schoar, and Wang (2008) investigate several underlying factors that drove high returns for university endowments from 1992 to 2005, including the size of the endowment, the quality of the student body, and the use of alternative assets. Brown, Garlappi, and Tiu (2010) study the contribution of asset allocation decisions (both long-term policies and tactical short-term deviations) and security selection within asset classes to the investment returns of endowment funds. Ang, Ayala, and Goetzmann (2018) demonstrate that university endowments have been shifting their investments from standard asset classes like stocks and bonds into alternative asset classes, such as private equity and hedge funds. More recently, Lo, Matveyev, and Zeume (2021) and Dahiya and Yermack (2021) use the tax filings of nonprofit organizations to study investment returns across a wide range of nonprofit organizations.[6]

Finally, we contribute to the literature on endowment risk aversion. In investigating the relationship between asset allocation and risk budgeting, Brown and Tiu (2010) suggest that most university endowments possess similar levels of passive risk, based on their asset allocation. Dimmock (2012) shows that university endowments that face riskier incomes will allocate a higher fraction of their endowments to safer assets, thereby hedging their risks. Brown, Dimmock, Kang, and Weisbenner (2014) show that endowments decrease their spending following large declines in endowment value, thus adjusting their stated long-term spending policies.

MODELING SPENDING POLICIES AND ENDOWMENT DYNAMICS

We begin by documenting five major spending policies commonly used by endowment funds in the United States: three variations of the Tobin spending rule, a "flat" spending rule, and a bounded adjustment rule. Each spending policy provides a basis for the amount of capital the endowment will earmark for spending in the next fiscal year. We express the total endowment value and spending percentage in year t under spending rule i as $W_{i,t}$ and $s_{i,t}$ respectively. The spending percentage $s_{i,t}$ is the fraction of endowment value $W_{i,t}$ spent in year t.

[5] See also Milevsky and Robinson (2005) for an early precursor work that studies sustainable spending rates for endowments with highly diversified portfolios. The paper concludes that payout rates should be lower than what is typically advised.

[6] See Cejnek, Franz, Randl, and Stoughton (2013) for an extensive survey of the literature on endowment funds.

We build a dynamic model that we use to simulate investment returns from a portfolio of assets. Our model contains two state variables, the annual endowment value and the spending amount. Asset returns are modeled by a multivariate lognormal distribution, with each asset j having a mean μ_j and variation σ_j^2. We estimate the parameters for the lognormal distribution by applying logarithmic transformation equations to historic investment return data for major asset classes. The Appendix contains information on these asset returns, their standard deviations, and the variance-covariance matrices.

We use this model to simulate the endowment value, spending percentage, and breakeven return values under each spending policy. We normalize the starting endowment value at $100 million, and use a 20-year time horizon. We evaluate the endowment performance using a number of measures described below to provide insights that may inform the development of spending policies.

In this section, we discuss the five major spending rules used by endowment funds. Most endowments also include an inflation adjustment as part of their spending rules. We model inflation as a constant time-invariant parameter. We use the following function to model the effects of inflation on spending rules:

$$infl(s_t) = s_t \times (1 + i(t))$$

where s_t is the spending amount without inflation in the current year, and $i(t)$ is the projected inflation percentage for the current year calculated or assumed from historical values. In our simulations, we fix inflation at 2%.

The 80/20 Tobin Rule

One of the most common endowment spending policies, named after Tobin (1974), is the 80/20 Tobin rule. Most notably, it is used by the endowment fund of Yale University.[7] Yale's Office of Financial Planning and Analysis states that, "the university adopted a policy specifically designed to stabilize annual spending levels and to achieve intergenerational neutrality by preserving the real value of the endowment portfolio over time."[8] As such, the Tobin rule incorporates both stability and market volatility into its calculation. Yale's overall spending policy is described in Swensen (2009) as follows: "spending for a given year equals 80 percent of spending in the previous year plus 20 percent of the long-term spending rate applied to the endowment's market level at the previous fiscal year end. The resulting figure is brought forward to the current year by using an inflation adjustment. Since previous levels of spending depend on past endowment market values, present spending can be expressed in terms of endowment levels going back through time. The resulting lagged adjustment process averages past endowment levels with exponentially decreasing weights."

In the market term of the spending equation, which receives 20% of its total weight, a long-term spending rate of 5.25% is applied to the endowment market level at the end of

[7] It is also commonly referred to as the "Yale model." See Tobin (1974) for further details.

[8] For further details, refer to "Introduction to the Endowment Spending Policy," Office of Financial Planning and Analysis, Yale University, September 2020.

the previous fiscal year. The inflation adjustment is then applied to the entire amount. The spending percentage in year t can thus be written as

$$s_{1,t} = \frac{infl\left(0.8 * (s_{1,t-1} * W_{1,t-1}) + 0.2 * \left(0.0525 * W_{1,t}\right)\right)}{W_{1,t}}.$$

Flat Spending Policy

A flat spending policy is characterized by spending a fixed percentage of the endowment value each year. One of the largest endowments that employs a flat spending rule is that of Stanford University. Each year, it spends 5% of the previous year's endowment value.[9] We consider this rule as a baseline since it is uncorrelated to earlier spending or market conditions. The inflation adjustment is applied to the spending amount. The rule therefore can be described by the following function:

$$s_{2,t} = \frac{infl(0.05 * W_{2,t})}{W_{2,t}}.$$

70/30 Adjusted Tobin Rule

Another common spending rule is the adjusted Tobin rule. It is modeled after the Tobin rule, most commonly employing 70/30 or 80/20 weights. The 70/30 adjusted Tobin rule is most prominently used by the University of Pennsylvania's endowment. In its annual financial report, the University of Pennsylvania describes its endowment spending policy as follows: "for fiscal year 2020, the spending rule target payout was based on the sum of: (i) 70% of the prior fiscal year distribution adjusted by an inflation factor; and (ii) 30% of the prior fiscal year-end fair value of the AIF, lagged one year, multiplied by 5.0% for all funds."[10] The Associated Investments Fund (AIF) is managed by the University of Pennsylvania's Office of Investments, so its value is treated as the current market value of the endowment.

It is important to note that, unlike the regular Tobin rule, where the inflation adjustment is applied to the entire spending amount, in the adjusted Tobin rule, the inflation adjustment applies only to the prior year's spending. The rule can be summarized by the following equation:

$$s_{3,t} = \frac{infl\left(0.7 * \left(s_{3,t-1} * W_{3,t-1}\right)\right) + 0.3 * (0.05 * W_{3,t})}{W_{3,t}}$$

According to its investment reports and press releases, Harvard University's endowment also appears to use the 70/30 adjusted Tobin rule as its spending policy.

80/20 Adjusted Tobin Rule

The 80/20 adjusted Tobin rule is similar to the 70/30 rule, but puts a higher weight on the previous year's spending. Before 2008, the Massachusetts Institute of Technology

[9] "The Stanford Endowment Fact Sheet," Stanford University, October 2019.

[10] University of Pennsylvania, Annual Financial Report, 2019-2020.

(MIT) used a variation of a flat spending rule, which "buffered spending volatility by averaging changes in endowment value over a three-year period and by targeting a distribution rate that varied between 4.75% and 5.5% of that average." Currently, however, MIT uses the 80/20 adjusted Tobin rule as its spending policy, with a 5.1% long-term spending rate.

The rule is described by the following equation:

$$s_{4,t} = \frac{infl\big(0.8 * (s_{4,t-1} * W_{1,t-1})\big) + 0.2 * (0.051 * W_{4,t})}{W_{4,t}}$$

Note that the inflation adjustment is applied only to the 80% weighted term.

Within an Acceptable Band

The final frequently used spending rule we consider is the "within an acceptable band" rule. We illustrate this rule based on a university endowment that preferred to remain anonymous. Under this rule, the previous year's spending percentage is brought forward to the current year using an inflation adjustment. The university then observes whether the inflation-adjusted spending rate falls within an established acceptable band, which ranges from 4% to 6.25%. If the spending rate falls within this band, then the calculated rate is used as this year's spending percentage. If the spending rate falls outside of this band, then the trustees of the university qualitatively determine a reasonable spending rate for the coming year that does fall within the acceptable band.

We assume that if the inflation-adjusted spending percentage falls outside the acceptable band, we adjust the spending percentage to either the lower or upper end of the band, which are 4% and 6.25%, respectively.

The rule is written mathematically as:

$$\text{If } \frac{infl(s_{5,t-1} \times W_{5,t-1})}{W_{5,t}} < 0.04, \qquad s_{5,t} = 0.04$$

$$\text{If } \frac{infl(s_{5,t-1} \times W_{5,t-1})}{W_{5,t}} > 0.0625, \qquad s_{5,t} = 0.0625$$

$$\text{If } \frac{infl(0.04 \times W_{5,t-1})}{W_{5,t}} \leq s_{5,t} \leq \frac{infl(0.0625 \times W_{5,t-1})}{W_{5,t}}, \qquad s_{5,t} = \frac{infl(s_{5,t-1} \times W_{5,t-1})}{W_{5,t}}$$

PERFORMANCE MEASUREMENT

To assess and quantify the performance, spending, and growth of these endowments, we use four specific metrics described in this section.

Endowment Growth

Perhaps the simplest metric to consider is whether the endowment is growing or shrinking, i.e., the average percent change in endowment value from the previous to the current year, W_{t+1}/W_t. A value greater than 1 signifies endowment growth for the year, while a value less than 1 signifies a decline in endowment for the year. In addition to reporting the average change in endowment value, it is also important to consider a range of stochastic possibilities. Thus, in addition to the mean and median, we document the 5th, 25th, 75th, and 95th percentiles for both the endowment and spending values in the Online Appendix.

Breakeven Return Value

A fundamental challenge for endowments is producing a sizable return that will offset spending and thus sustain endowment growth. This breakeven return value, or b, is simply the minimum return percentage that engenders a nonnegative change in the cumulative endowment value:

$$W_{t+1} = b_t * (W_t - s_t) > W_t.$$

The breakeven return value depends on the spending policy chosen by the endowment. In the Online Appendix, we derive breakeven return values for each of the five spending rules.

Spending vs. Endowment Growth

We use the relative change to quantify the marginal increase in spending per percent gain in endowment value. It is defined as

$$\frac{avg\left(\frac{s_t - s_{t-1}}{s_{t-1}}\right)}{avg\left(\frac{W_t - W_{t-1}}{W_{t-1}}\right)}$$

across all values of t.

Intuitively, a value of 1 implies that spending is changing at the same rate as the endowment value. A value of -1 implies that spending is increasing at the same rate that the endowment value is decreasing (or vice versa). A positive value with a magnitude greater than 1 indicates that spending is increasing more than the endowment value is increasing. This implies that for every dollar of growth in the endowment, the university will increase its spending by more than a dollar, which may not be sustainable in the long-term. Similarly, a negative value with magnitude greater than 1 means that the spending rate is decreasing at a faster rate than the endowment value is increasing (or vice versa). We note that the relative change is averaged across all simulated years, and not only the average simulated path.

We also consider a benchmark spending metric. It focuses on a benchmark value for the annual spending percentage, which we take to be 5%, as most endowments anchor themselves around this amount. The metric measures the percent deviation from the anticipated benchmark value, computed as $(s_t - 0.05)/0.05$.

Worst-Case Scenario Metrics

Finally, we employ a number of metrics to measure worst endowment outcomes. First, we measure the single year of largest loss in endowment value over a bounded time frame. This is used to understand the annual tail-end risks around short-term endowment decline. We analyze both the average annual largest loss across all simulations, and additionally, the single greatest loss in any year, to determine the expected and worst-case losses.

To further study the risk of endowment loss, we measure the maximum drawdown, the largest continuous reduction in endowment value from peak to trough over a chosen time horizon. The maximum duration is the period in years over which this loss occurs. This is useful for managers in evaluating the potential long-term risk. It is computed for the endowment as

$$\frac{\min(W) - \max(W)}{\max\,(W)}$$

and the spending value as

$$\frac{\min(s) - \max\,(s)}{\max\,(s)}.$$

As with the largest loss, we analyze the average maximum drawdown across all simulations and the peak maximum drawdown across any time period of the simulation. We also measure the frequency and duration of the simulated endowments reaching ruin, defined as the complete loss of funds.

RESULTS

Our goal is to examine how much in actuality endowments must spend in order to finance their annual operations and to sustain their future growth. To achieve this goal, we study how different spending rules affect growth in endowment value, spending percentages, and breakeven return values over time. We briefly analyze edge cases and equal allocations for both sets of the asset classes under consideration. We then examine the asset allocations of the largest higher education endowment funds (Harvard, Yale, Stanford, and MIT), and study their performance under five different spending policies.

Asset Classes

To generate realistic endowment simulations and emulate the exact asset allocation of major endowments, we rely on asset class data from two sources: the Almanac of Returns Data: Stocks, Bonds, Bills, and Inflation (SBBI), and aggregated data from frequently invested indexes in equity markets, fixed income, hedge funds, private equity, real estate, and real assets (Common).

The SBBI has six asset classes (outlined below) and includes monthly return data from 1926 to 2018:

1) Large Cap Stocks

2) Small Cap Stocks (SCS)
3) Long-term Corporate Bonds (LCB)
4) Long-term Government Bonds (LGB)
5) Intermediate Government Bonds (IGB)
6) U.S. 30-Day Treasury Bills (UTB)

We use monthly returns over varying periods (depending on the length of time that the series has been tracked) for the Common asset classes, which include the following:

1) Domestic Equity: Russell 3000
2) International Equity: MSCI World IMI Index (USD)
3) Emerging Markets Equity: MSCI ACWI EM Investable Market Index (IMI)
4) Fixed Income: Dow Jones Corporate Bond Index
5) Hedge Funds: Barclays Hedge Fund Index
6) Private Equity: Average Returns of The Blackstone Group (BX), The Carlyle Group (CG), KKR & Co., Inc. (KKR), and Apollo Global Management (APO)
7) Real Estate: S&P Global REITs
8) Real Assets: S&P Real Assets (RA) Index

Edge Cases and Equal Allocation

The two sets of asset classes in our study serve different, yet equally important, purposes. The first set of asset classes (SBBI) comprises broad asset classes that add context to the general trends in endowment investing. The second set of asset classes (Common) in which university endowments commonly invest allows us to simulate the investment returns of the current portfolios of several top-performing university endowments. We first explore edge cases, undiversified portfolios in which there is a complete allocation in a single asset class, to determine the risk-reward tradeoff of each asset class, since the universities in our study place a large emphasis on asset allocation decisions to meet their target returns and spending percentages each year. We also formulate a baseline portfolio that has an equal allocation across all asset classes in the Online Appendix.

The highest performing SBBI edge case is a homogenous investment in small cap stocks; it grows to an endowment value of almost $800M over our simulated time horizon of two decades, and possesses the highest annualized mean return at 16.1%.[11] Three other edge case allocations induced positive, albeit significantly smaller, endowment growth: large cap stocks (a terminal value of $400M), long-term corporate bonds (a terminal value of $125M), and long-term government bonds (a terminal value of $115M). Investing completely in either intermediate government bonds or U.S. 30-Day Treasury bills leads to endowment ruin over this two-decade period. This ruin occurs because the annualized mean returns are not sufficient to offset the spending each year. An equal allocation to all SBBI asset classes yields a terminal endowment value of about $180M.

Of the eight Common asset classes, several yield highly profitable results, including real assets, private equity, domestic equity, and an equal allocation between all eight asset classes. We note that a 100% investment in domestic equity produces a similar endowment

[11] Refer to the Online Appendix for the details.

growth pattern to an equal asset allocation, since the mean returns of the portfolio are roughly the same (see the Online Appendix). However, the equal allocation portfolio possesses increased robustness due its implicit diversification, and is thus a preferable mechanism to spread idiosyncratic risk. Less profitable asset classes include emerging markets equity (a terminal value of $260M), real estate (a terminal value of $240M), international equity (a terminal value of $220M), hedge funds (a terminal value of $180M), and fixed income (a terminal value of $130M). None of the common asset class edge case portfolios lead to ruin.

The Effect of Spending Rules on Future Endowment Value and Spending

Using disclosures from annual reports, Exhibit 1 documents the asset allocations of Harvard, Yale, Stanford, and MIT's endowments. We use these allocations to compare the endowment trajectories under these different spending rules.

Harvard's asset allocation is based on the Harvard Management Company's FY2019 Financial Report.[12] Our simulation results show that Harvard's endowment will continuously grow over the next two decades. The paths for endowment value growth (Exhibit 2, Panel A) are relatively similar for each spending rule, displaying exponential growth to approximately between $375 million and $425 million. Using a flat 5% spending policy (SR 2) results in the lowest endowment value over time. This is consistent with the trend we observe in spending curves (Exhibit 2, Panel B), as SR 2 requires spending at a flat rate of 5.1% (inflated), while the other rules lead towards a decreasing spending rate.[13] On the other hand, the Within Acceptable Band rule (SR 5) yields the lowest final spending percentage, approximately 4.2% at the end of the simulation horizon.

Harvard spent 5.1% of its endowment in FY 2019, corresponding to a breakeven return value of 5.3%, significantly higher than the standard university return target of 5% per year (Exhibit 2, Panel C). This indicates a need for Harvard to reevaluate its asset allocation to grow the endowment more effectively, and in turn sustain a higher long-term level of spending.

Yale's asset allocation is based on the Yale Investment Office's fiscal year 2020 allocation targets (Exhibit 1). Many of Yale's returns come from absolute return, which we classify under "Hedge Funds." Our simulation shows that Yale's endowment also exhibits continuous exponential growth under all spending rules (Exhibit 3, Panel A), resulting in a value of approximately $450 million to $525 million after two decades. The flat 5% spending policy (SR 2) results in the lowest endowment value, since this rule maintains the highest spending percentage over time (Exhibit 3, Panel B). The 80/20 Adjusted Tobin Rule (SR 4) results in the highest terminal endowment value.

[12] Due to rounding, the cumulative reported allocation exceeds 100%, so we reduce cash from 2 to 1% as a manual correction. Additionally, since Harvard reports asset classes outside of our classification, we categorize Natural Resources and Other Real Assets under "Real Assets," and Bonds/TIPS and Cash under "Fixed Income."

[13] Note that a decrease in spending rate (as a fraction of endowment value) does not imply a decrease in the amount of spending.

In FY 2019, Yale spent approximately 4.5% of its endowment value, markedly lower than Harvard's spending percentage that year. This spending percentage is also consistent with the range of our projected spending levels under all spending policies other than the flat 5% (Exhibit 3, Panel C). This result holds even when Yale uses its own spending rule, the 80/20 Tobin Rule (SR 1). Thus, Yale's endowment seems well positioned to sustain growth and support university operations under its current spending rule and asset allocation.

Stanford's asset allocation is based on the Stanford Management Company's FY2019 Investment Report (Exhibit 1). As before, we classify absolute return under "Hedge Funds." Stanford's endowment exhibits consistent growth for all spending rules from approximately $400 million to $475 million (Exhibit 4, Panel A). The flat 5% spending policy (SR 2) produces the lowest terminal endowment value, while the 80/20 Adjusted Tobin Rule (SR 4) results in the maximum endowment value, which inversely mirrors the trend for spending percentage (Exhibit 4, Panel B). Stanford's breakeven return values are very similar to those of Yale, under 5% for all rules except SR 2 (Exhibit 4, Panel C).

In its FY 2019 Investment Report, Stanford acknowledges that its endowment has dedicated the past several years to reducing the number of active partners within each asset class for the Merged Pool Investment Portfolio. This process involved liquidating a long tail of investments, which hindered past endowment performance and may still negatively affect returns into the near future. This may particularly apply when the university must maintain a high spending percentage to enable a "robust annual disbursement to the current operating budget."[14] Nonetheless, Stanford believes a more selective portfolio in the long term will be worth the losses from this short-term restructuring.

MIT's asset allocation is based on the MIT Investment Management Company's FY2019 Investment Report (Exhibit 1). Our simulations show that MIT's endowment will grow over the next two decades to approximately $375 million to $425 million, depending on its choice of spending rule (Exhibit 5, Panel A). Because of its non-decreasing percentage of spending, the flat 5% spending policy (SR 2) results in the lowest endowment value over time. The 80/20 Adjusted Tobin Rule (SR 4) yields the highest terminal endowment value, even though the lowest spending percentage in Year 20 originates from the Within Acceptable Band rule. MIT's breakeven return values are very similar to those of Harvard (Exhibit 5, Panel C).

In FY 2019, MIT's endowment spent approximately 4.3% of its pooled funds, the lowest spending rate of the four named universities under consideration. However, even with the most conservative spending rate of this group and normalizing for its initial endowment size, MIT does not reach as high of a terminal endowment value as, for instance, Yale. This may be due in part to MIT's emphasis on equity-based investments, while Yale allocates over a third of its endowment to private equity, which yields higher average returns in our model. Its large equity investments allow MIT to take on less risk than Yale does while still earning significant returns.

[14] Disclosure on Stanford's Merged Pool concentration strategy can be found at https://smc.stanford.edu/wp-content/uploads/2020/01/Stanford-University-Investment-Report-2019.pdf

Overall, different spending rules do not greatly impact the overall university endowment growth trajectory. Because the annual spending allocation falls typically under or around 5% in our simulations, the generation of returns dominates the endowment's path. The additional volatility in returns, compared to the predictability in spending, the deterministic nature of these policies, and the reflexive changes in spending based on the returns of prior years, further highlight this point. However, the exception to this trend is SR 2, which prescribes a flat spending percentage, and thus does not account for past or present endowment value. In our simulations, we found that when SR 2 is used by an endowment, the terminal endowment value after two decades is $50M lower than the endowment values generated by comparable policies.

We find there are statistically significant differences in spending percentage curves when analyzing the different policies. SR 2, the flat 5% rule adjusted for inflation, remains constant at 5.1% after year 2, as expected. For all other SRs, spending experiences a roughly exponential decline over the first decade, and converges to an asymptotic value sometime between years 10 and 14, depending on the school. The 80/20 Tobin Rule (SR 1) and the 70/30 Adjusted Tobin Rule (SR 3) plateau faster than the 80/20 Adjusted Tobin Rule (SR 4) and Within Acceptable Band (SR 5). We note that a decline in the spending percentage does not correlate with a decline in the amount spent; it merely means that the growth in the amount of spending (or need) by a university is less than the overall growth of the endowment. In fact, this can be seen as a necessary condition for a robust endowment, although yearly deviations from this trend due to high-cost special projects or initiatives are acceptable, and should not affect its long-term outlook.

Key Attributes of Each Spending Policy

There are three fundamental factors that influence how each Tobin rule determines the spending percentage: its benchmark spending rate for the market volatility term, its treatment of inflation, and its weighting split between stability and market volatility terms. As described, Yale and MIT assign an 80% weight to the previous year's spending and a 20% weight to the benchmarked percentage of this year's endowment, but they use different benchmarks (Yale at 5.25%, MIT at 5.1%) and inflation adjustments. UPenn and Harvard, on the other hand, assign 70% and 30% weights to these terms.

In analyzing the relative importance of these factors, our results demonstrate that the most significant marginal impact on the spending percentage results from the weighting split, followed by the benchmark and the inflation treatment, which nonetheless still possess a noticeable effect. To assess the impact of the weighting split, we examine the differences between the 70/30 Tobin Rule (SR 3) with a 5% benchmark and the 80/20 Adjusted Tobin Rule (SR 4) with a 5.1% benchmark. Across all four university simulations, SR 3 yields both a higher spending percentage over the two-decade period and a higher asymptotic spending percentage than SR 4. When we normalize the benchmark at 5%, we notice the same trend, but the difference in spending is even further pronounced. The weighting split is thus the driving factor differentiating these rules. This is reasonable, as spending percentage paths decline over time; therefore, the higher weight of the benchmark percentage in SR 3 (30%), greater than any annual allocated percentage on these paths, will produce a larger spending percentage in the current year.

The effect of the benchmark percentage is best illuminated through a comparison of the 80/20 Tobin Rule with a 5.25% benchmark (SR 1) and the 80/20 Adjusted Tobin Rule with a 5.1% benchmark (SR 4). In the simulations for all four universities, SR 4 can be seen as a vertically shifted and scaled transformation of SR 1. When we normalize inflation by inflating the entire Tobin rule equation, rather than only the 80% weighted element (as in SR 4), we notice a similar trend, but the difference in spending percentage is slightly smaller, displaying the marginal effect of the benchmark exclusively.

We note a similar effect for the inflation treatment when analyzing the differences between the 80/20 Tobin Rule with a 5.25% benchmark (SR 1) and the 70/30 Tobin Rule with a 5% benchmark (SR 3). SR 1 yields both a higher spending percentage over the two-decade period and a higher asymptotic spending percentage than SR 3, while the vertical difference between the two curves remains relatively constant from Year 7 onwards. As observed above, the higher weighting split in SR 3 (30%) to the benchmark percentage drives a larger spending percentage overall. However, when we consider all three factors (weighting split, inflation treatment, and benchmark percentage), SR 1 has both a larger inflation adjustment and a larger benchmark. Since we observe that SR 1 has a slightly higher spending percentage per year overall, but SR 3 produces a higher spending percentage based solely on its weighting split, we can conclude that these results are due to the higher inflation treatment and benchmark percentage in SR 1. Thus, while the weighting percentage is the most influential factor in determining the spending percentage by itself, it can be outweighed by the combination of the other two factors in the Tobin rule.

Comparing the Major Endowments

To further understand the impact of spending policies on endowments, we present additional evaluation metrics—relative change, average change in endowment value, benchmark spending analysis, largest loss, and maximum drawdown—applied to university asset allocations.

Exhibit 6 shows the relative change, average change, and benchmark spending for all 4 endowments and 5 spending rules.[15] Under its own spending policy (80/20 Tobin Rule, SR1), Yale has a relative change value of 1.1. Yale and Stanford have relative change values that are positive and slightly greater than 1, which implies that Yale and Stanford's spending percentages change at approximately the same rate as the value of the endowment. On the other hand, Harvard and MIT have relative change values that are negative and significantly smaller in magnitude than 1, which implies that each endowment's spending percentage decreases at a faster rate than the endowment value increases per year.

Exhibit 6, Panel B shows that the average annual change in endowment value over time is highest for Yale, with a value of 8.6%. Harvard, Stanford, and MIT's endowment values change at about the same rate as in our simulations. Exhibit 6, Panel C shows the benchmark spending. Since the benchmark spending measures the percent deviation of the endowment's actual spending percentage from the university's initial target (usually 5%), the SR2 benchmark is constant for each university, and only incorporates compounded

[15] Spending rule 1 (SR1) is Yale's actual spending rule, SR2 is Stanford's spending rule, SR3 is Harvard's and SR4 is MIT's.

inflation. Based on our simulation results, Harvard, Yale, and MIT all spend well below the 5% benchmark, as shown by the negative values.

Next, we examine the maximum largest loss, the paramount annual loss across any year in the simulations. This is considered an indicator of the worst-case scenario for annual endowment performance (Exhibit 7, Panel A). We observe that using the flat 5% rule (SR 2) results in the smallest maximum largest loss for each endowment, while either the 80/20 Tobin or adjusted Tobin rules (SRs 1 or 4) yield the highest loss. Yale stands to lose over twice as much in a given year in its worst-case scenario as does Harvard, while Stanford and MIT fall in between those two schools in risk profile. This finding is consistent with our knowledge that Harvard is exposed to fewer risky assets (i.e., they are not exposed to much venture capital) than the other endowments. Thus, while Yale, Stanford, and MIT could potentially lose more money than Harvard in a given year, they may perform better overall due to increased exposure to risky asset classes, which we observe through higher peak endowment values for Yale, Stanford, and MIT in our simulations.

We then studied the average largest loss in endowment value (Exhibit 7, Panel B) of the annual largest loss for each 20-year trajectory. This is computed by taking the mean across Monte Carlo simulated paths, and reveals the largest decline an endowment is expected to have in any one year over a substantial time horizon. Yale is expected to have the worst endowment decline in a single year, possessing an average largest loss of $50 million. Stanford is a close second with a value of $40 million, then MIT at $30 million, and finally, Harvard at $25 million. Spending policy has less impact on the average largest loss than on the maximum largest loss.

Exhibit 8 shows the largest and average maximum drawdowns and their corresponding duration. Similar to the trends we observed for largest loss, we note that Yale has the highest value for “max,” the maximum drawdown, as it reaches nearly $700 million (meaning there was a time frame during which the Yale endowment experienced an overall reduction of $700 million in its endowment value). Harvard’s largest maximum drawdown is about half that value, reaching around $300 million in its worst performing years (Exhibit 8, Panel A). Interestingly, Harvard has a maximum drawdown duration of 6 years between its peak and trough, while Yale, Stanford, and MIT have maximum durations of between 3 to 4 years (Exhibit 8, Panel B), indicating that Harvard would experience a more gradual decline.

Regarding the average maximum drawdown, we witness similar relative trends between universities, but at much lower magnitudes of prolonged loss. Drawdown values also appear to be consistent across spending rules (Exhibit 8, Panel C). We note that, on average, each endowment experiences approximately equivalent average maximum drawdown periods of 2 to 3 years (Exhibit 8, Panel D). This finding is particularly useful for endowments that may be looking to accurately predict the duration of longer-term losses.

Under our simulated paths for Harvard, Yale, Stanford, and MIT’s endowments, no endowment experienced total ruin over a 20-year horizon. This is a reasonable result, since the allocations are diversified and de-risked to prevent catastrophic losses, except in the case of market crashes or "black swan" events, neither of which are incorporated in our simulation model.

University Endowments vs. Optimized Portfolios

In order to gauge levels of endowment risk aversion, which we use in our subsequent analysis, we compare their asset allocations to that of commonly used financially optimized portfolios. In particular, we compute the minimum variance portfolio, the maximum Sharpe ratio portfolio, and the mean-variance tradeoff portfolio.

Exhibit 9 juxtaposes the ideal asset allocations under these three different optimizations with the allocations of the four universities. The maximum Sharpe ratio portfolio is heavily weighted toward fixed income (53.3%) and hedge fund (40.9%) index investing. Both of these asset classes possess the smallest annualized standard deviation by a significant margin, despite having modest annualized returns compared to asset classes such as private equity or real assets. Real estate receives a noticeable allocation of 5.3%, due to its low covariance with fixed income and hedge funds, even though other asset classes have lower standard deviations. The minimum variance portfolio is nearly identical to this, which corroborates the Sharpe portfolio's evaluation of minimizing risk relative to maximizing return.

We compute an additional mean-variance efficient portfolio by optimizing the portfolio allocation for the difference between expected return and the risk-aversion weighted variance:

$$\begin{cases} \underset{h}{\text{Maximize}} & r^T h - \lambda h^T \Sigma h \\ \text{subject to } e^T h = 1 \end{cases}$$

where h represents an $n \times 1$ vector of asset weightings, r is an $n \times 1$ vector of asset returns, Σ is the $n \times n$ covariance matrix, and λ is a nonnegative risk aversion parameter. The λ value is positively correlated to increasing risk aversion, as higher values lead to greater penalization of the variance term. The result of this optimization yields the mean-variance efficient portfolio for a given parameter of risk aversion.

The mean-variance tradeoff portfolio, with a λ of 2, consists of about half domestic equity and between 7 to 15% of hedge funds, private equity, real estate, and real assets. The predominant allocation to domestic equity is sensible under the objective function, since the domestic equity index dwarfs all asset classes in terms of annualized mean return (0.1339) except for private equity and real assets, yet possesses a significantly lower annualized standard deviation (0.1545) than private equity (0.3598) or real asset indexes (0.5681). Hedge funds and real estate have low covariances to domestic equities compared to those of non-invested asset classes like emerging markets, which have a marginally higher annualized mean return.

To compare these optimized portfolios to the actual allocations of major endowments, we show their root-mean-squared errors in Exhibit 10. Yale demonstrates the greatest differences in its allocation to all three optimized portfolios, likely due to the riskier nature of Yale's portfolio. The most significant results are the parallels between Harvard's portfolio and the mean-variance tradeoff portfolio. Both place significant investment into domestic equities compared to other universities or optimized portfolios. Harvard engages more in hedge fund and private equity investment and less in real estate and real asset

investment than the mean-variance tradeoff portfolio, but their overall risk profiles in aggregate nevertheless have very similar presentations.

Endowment Investing vs. Spending

Finally, we study the interdependence between endowment investing and spending. We start by using the Merton (1993) endowment portfolio choice model to estimate a frontier of risk aversion and marginal propensity to increase endowment wealth when spending needs change.

In the model, a representative nonprofit endowment solves its portfolio optimization problem while having a flow of income from non-endowment sources (e.g., private donations or public funding). Merton (1993) demonstrates that the optimal investment portfolio hedges the risk of cash flows from these non-endowment sources. The model can be solved in closed form to yield the following optimal portfolio allocation, h:

$$h_i W = A b_i + \sum_{k=1}^{m} D_k d_{ki}\,, \qquad i = 1, \ldots, n$$

where $b_i = \sum_{j=1}^{n} v_{ij}(\alpha_j - r)$, $d_{ki} = \sum_{j=1}^{n} \sigma_j g_k S_k \eta_{kj} v_{ij}$, A is the reciprocal of the absolute risk aversion, and D_k depends on university intertemporal preferences, measuring the university's marginal utility in regard to expenditures and net worth. In terms of parameters, i and j represent asset classes, k represents a university activity, v is the inverse covariance matrix for assets, α_j is the expected annualized return for asset j, r is the risk-free rate, σ_j is the standard deviation of asset j's return, g_k is the standard deviation of the growth rate of activity k's cost, S_k is the unit cost of bankrolling activity k, and η_{kj} is the correlation coefficient between Wiener processes for the asset return dynamic and the activity expenditure dynamic. In our analysis, we simplify this formulation by not subdividing university expenditure into different activity designations.[16]

For each endowment, we use its data on asset allocation and perturb a range of absolute risk aversion values, for each value then regressing the D_k parameter to approximate the intertemporal preference of the university towards marginal wealth and spending. Exhibit 11 shows a frontier of absolute risk aversion, $1/A$, and the marginal utility of wealth per unit increase in spending, D_k. At all levels of absolute risk aversion, the four universities exhibited a positive D_k value, which implies that their desire to increase the endowment is correlated with a rise in annual spending. This is an intuitive result, as any nonprofit organization spearheading a high-cost special initiative may pressure its endowment to generate a higher than usual annual return on investment to replenish it.

We can additionally interpret D_k as a measure of relative independence between the desire for endowment growth and endowment spending. A large magnitude of D_k indicates that investment decisions and asset allocation must be strongly dependent on changes in spending. Normalizing for absolute risk aversion, it is evident that Harvard and Stanford have the greatest independence, and are thus more likely to adjust their investment strategy based on changing operational needs of the university or exogenous events, such as the

[16] We take $h_i W = A b_i + D_k d_{ki}$, where k represents the cumulative spending of the university

COVID-19 pandemic. Yale and MIT, on the other hand, demonstrate a lower marginal utility towards gaining wealth conditioned on changing spending.

Harvard and Stanford's greater interdependence between spending and the state of their endowment growth (from both investment returns and university-generated revenue) can be witnessed in their responses to the COVID-19 pandemic. Due to revenue loss, Harvard reduced its spending on capital projects and acquisitions from $903 million in 2019 to $627 million in 2020. Stanford, meanwhile, adopted a slightly riskier investment portfolio that ended up producing better than expected returns in an effort to mollify their 19% decline in their 2020 income.

Since we can deduce the level of risk aversion from our prior empirical simulation analysis and qualitative trends in university investment reports, we thus remove the normalization from our analysis in order to assess each university's true interdependence between the state of their endowment growth and spending. Since Yale and Harvard can be characterized as the least and most risk-averse endowments, respectively, the preference for interdependence is very similar between Harvard, Stanford, and Yale. MIT, on the other hand, appears to possess a much smaller desire to acquire wealth in response to heightened costs (or lower revenue), or to de-risk an investment profile because of reduced spending. Relative to the other three schools, MIT prioritizes stability over adaptation.

CONCLUSION

Every endowment designs its spending rule to accomplish two competing objectives. First, endowments aim to support organizational operations and special initiatives through annual spending distributions. At the same time, endowments must maintain enough financial support for future needs, both known and often unknown. The most common approach to manage the tradeoff between these two objectives is to implement a long-term spending rate coupled with a smoothing rule to adjust its spending levels in any given year.

Many endowments model their spending rule based on the rule pioneered by Tobin (1974), which incorporates both a degree of stability and a component exposed to market volatility. For example, Yale University uses the original Tobin rule, which assigns 80% weight to the stable term of the previous year's spending and 20% weight to the volatile market value of the endowment. Other endowments take different approaches, for example, keeping a constant 5% spending rate (Stanford University) or using a different weighting for the two components of the original Tobin rule (Harvard University).

Apart from spending policy, we studied the effect of asset allocation on the risk-return dynamics of endowment investing. Since endowments must be inherently loss-averse in order to protect the financial wellbeing of the universities they serve, significant results stem from the phenomenon that a greater risk also implies a greater potential to experience higher losses in the short-term (measured by the largest loss) and the long-term (measured by the maximum drawdown). We compared the worst-case simulated single-year loss to the expected largest annual loss over a two-decade period, and found that, while the relative rankings between universities were the same, the spread was heightened with the worst-case losses. We discovered similar trends between the values and durations of the largest

loss and the maximum drawdown, implying that for these four universities, short-term and long-term loss behavior are not statistically different from one another. In regard to endowment ruin, we experience no complete loss of funds in our simulations, but it must be noted that we used lognormal simulations that do not adequately account for market recessions or black swan events. Ruin is still very unlikely for portfolios based on endowment allocations, but we caution that, despite our results, ruin is not an impossibility.

With respect to individual university endowments, Yale's expected annual growth is 1% greater than the other analyzed universities, but it faces a much steeper tail for annual losses. For the maximum largest loss, we observe that Yale stands to lose almost twice as much in an absolute worst-case scenario than Harvard. The riskier nature of Yale's portfolio is buttressed by the contrast of university asset allocation to optimized portfolios that penalize uncertainty: Yale demonstrated the highest deviation of all four universities in asset allocation from the minimum variance, maximum Sharpe ratio, and mean-variance tradeoff portfolios. This high-risk, high-return profile is consistent with Yale's allocation of one-third of its portfolio to private equity in 2020.

Conversely, our analysis indicates that Harvard has a lower variance asset allocation than the other universities studied. Harvard's portfolio possesses a very similar profile to the optimized mean-variance tradeoff portfolio: both place a large share of investment in domestic equity, and both maintain a non-negligible investment in high upside verticals (private equity for Harvard, real assets for the mean-variance tradeoff portfolio). Harvard also exhibited the smallest expected and worst-case losses both in the short-term and long-term. However, while Harvard draws down only half of Yale's dollar amount in its worst-case scenario, the duration lasts for six years, indicating that Harvard may have greater difficulty recovering from market downturns or unexpected loss events.

Stanford displays similar annual expected growth patterns to Harvard, its three largest categories of investment being international equity, hedge funds, and private equity. Stanford's expected two-decade largest loss and maximum drawdown is more in line with Yale's expected losses than Harvard's, which may be due to the heightened comparative allocation to riskier asset classes (private equity and real assets). However, the worst-case losses and drawdowns for Stanford are more similar to Harvard than to Yale, suggesting Stanford has a comparatively reduced tail-end risk. Likewise, MIT has a similar trajectory of its expected endowment, with a loss profile based on our simulations between Harvard and Stanford. (Interestingly, MIT invests almost twice as much in fixed income than the other schools.)

Finally, we explored the specific relationship between investing and spending for each endowment, both empirically (from our simulation results) and analytically (from Merton's endowment model). During market downturns or disasters, such as the COVID-19 global pandemic, universities must be prepared to allocate additional funds to meet the needs of the school. The investment performance of the endowment may also suffer in the years of the downturn. Therefore, analyzing the adjustments to spending based on changes in investment return, and analyzing the modifications to asset allocation (or overall investment strategy) based on changes in spending, are of utmost importance.

Based on our simulations, the marginal increase in spending per gain in endowment value—the relative change value—is positive for Yale and Stanford. Stanford has the highest relative change value at 1.3, followed by Yale at 1.1, although both values are close to 1, indicating that spending and endowment value are increasing at nearly the same rate. This is consistent with our knowledge that the Tobin rules used by Yale and Stanford are intended to smooth spending percentages so that they closely correspond to changes in endowment value. On the other hand, Harvard and MIT have negative relative change values, whose magnitudes are significantly smaller than 1. This indicates that spending is declining at a faster rate than endowment growth, which in turn may indicate a more conservative approach to maintaining endowment size. This finding is consistent with the investment strategies and asset allocations of both universities. In its 2019 investment report, Harvard claims to have developed a more risk-averse asset allocation intended to preserve the steady growth of its endowment. Similarly, MIT favors investing in equities to preserve growth, but stipulates a higher relative weighting of its stability term (80%) over its market volatility term (20%) to suppress spending jumps that could ensue from a high annual investment yield.

Merton's (1993) endowment model yields additional insights into the relative dependence between investment and spending decisions through the use of the absolute risk aversion and marginal utility towards wealth at the spending frontier. The endowments we examined invariably have a positive correlation between an increase in absolute spending (not spending percentage) and endowment growth, a foundational assumption of the Tobin rule. If we normalize for absolute risk aversion across all four universities, our results suggest that Harvard and Stanford have a larger desire than Yale and MIT to increase their endowment size pending a need for more spending, and vice versa for reduced spending.

We conclude by remarking that no spending rule, investment risk profile, nor university utility preference is objectively superior. It is imperative that each university first develop and intertemporally evolve their own objectives and preferences. Universities can then establish a spending rule, coupled with thoughtful asset allocation decisions, to advance their specific aims. As we have shown, the Tobin rule offers a high degree of customization through modifying the relative weights of the stability and market volatility terms, the benchmark percentage, and the treatment of inflation. Likewise, asset allocation presents an opportunity to imprint a chosen degree of risk aversion or risk tolerance on the endowment. The degree of interdependence between spending and investing lies at the core of each university's philosophical preferences, which can be manifested through the endowment's relative adaptability to exogenous events (market fluctuations, large-scale capital projects, or unforeseen revenue loss). While all four endowments differ in many of these regards, they nonetheless each have been successful in achieving their university objectives, and present themselves as valuable models for future endowment development and study.

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Exhibit 1
University endowment allocations across common asset classes

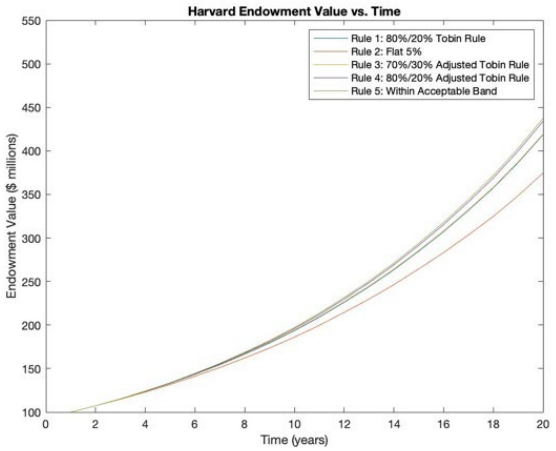
This table reports asset allocation data for the endowments of Harvard University, Yale University, Stanford University, and Massachusetts Institute of Technology (MIT). Data for Harvard University is obtained from its FY 2019 Financial Report. Data for Yale University is based on its FY 2020 target allocations obtained from a September 27, 2019 press release. Data for Stanford University is obtained from the Stanford Management Company FY 2019 Investment Report. Data for MIT is obtained from the Report of the Treasurer for FY 2019.

Asset Class	Harvard FY 2019	Yale FY 2020	Stanford FY 2019	MIT FY 2019
Domestic Equity	26.00%	2.75%	7.00%	11.00%
International Equity	-	13.75%	20.00%	22.00%
Emerging Markets (EM) Equity	-	-	-	-
Fixed Income and Cash Instruments	7.00%	7.00%	8.00%	16.00%
Hedge Funds	33.00%	23.00%	20.00%	13.00%
Private Equity	20.00%	38.00%	30.00%	23.00%
Real Estate	8.00%	10.00%	8.00%	15.00%
Real Assets	6.00%	5.50%	7.00%	1.00%

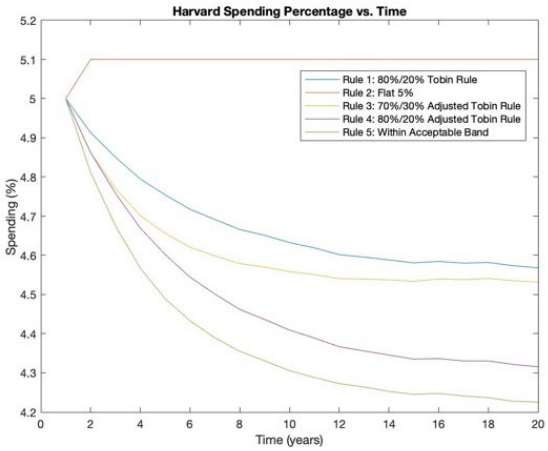
Notes: Harvard University's FY 2019 Financial Report is available here: https://finance.harvard.edu/files/fad/files/fy19_harvard_financial_report.pdf. Yale University's September 27, 2019 press release: https://news.yale.edu/2019/09/27/investment-return-57-brings-yale-endowment-value-303-billion. Stanford Management Company's FY 2019 Investment Report: https://smc.stanford.edu/wp-content/uploads/2020/01/Stanford-University-Investment-Report-2019.pdf. MIT's FY 2019 Report of the Treasurer: https://vpf.mit.edu/sites/default/files/downloads/TreasurersReport/MITTreasurersReport2019.pdf.

Exhibit 2
Simulated value, spending, and breakeven return for the endowment of Harvard University

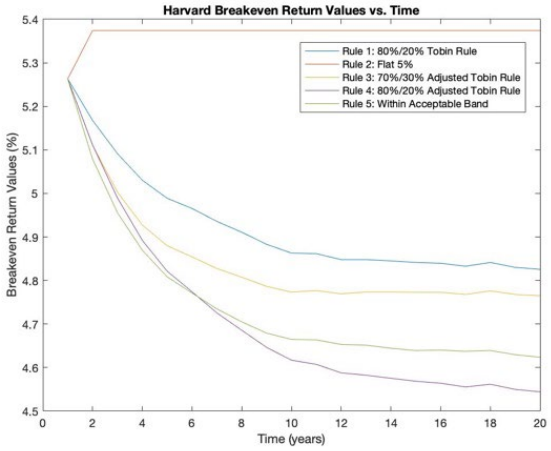
This figure shows simulated endowment value, spending percentage, and breakeven return for the endowment of Harvard University under five major spending rules. Rule 1 is the 80/20 Tobin rule, Rule 2 is the flat 5% rule adjusted for inflation, Rule 3 is the 70/30 adjusted Tobin rule, Rule 4 is the 80/20 adjusted Tobin rule, and Rule 5 is the within acceptable band rule.



Panel A: Endowment Value



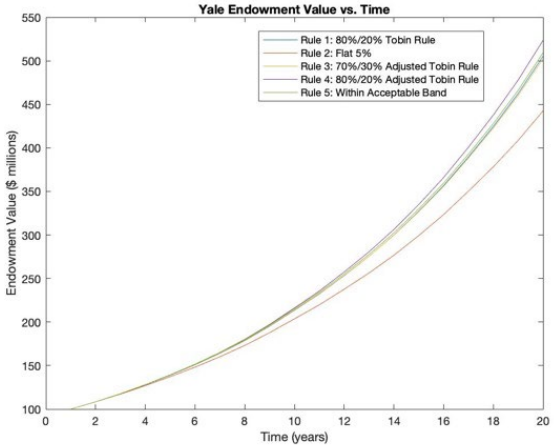
Panel B: Spending Percentage



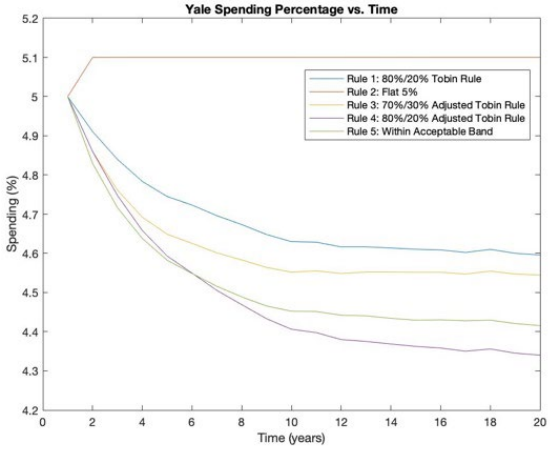
Panel C: Breakeven Return

Exhibit 3
Simulated value, spending, and breakeven return for the endowment of Yale University

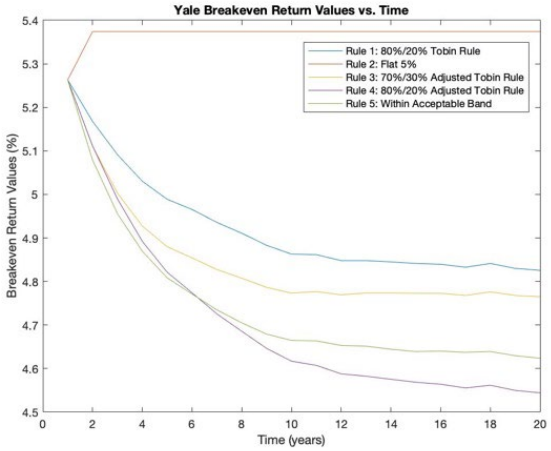
This figure shows simulated endowment value, spending percentage, and breakeven return for the endowment of Yale University under five major spending rules. Rule 1 is the 80/20 Tobin rule, Rule 2 is the flat 5% rule adjusted for inflation, Rule 3 is the 70/30 adjusted Tobin rule, Rule 4 is the 80/20 adjusted Tobin rule, and Rule 5 is the within acceptable band rule.



Panel A: Endowment Value



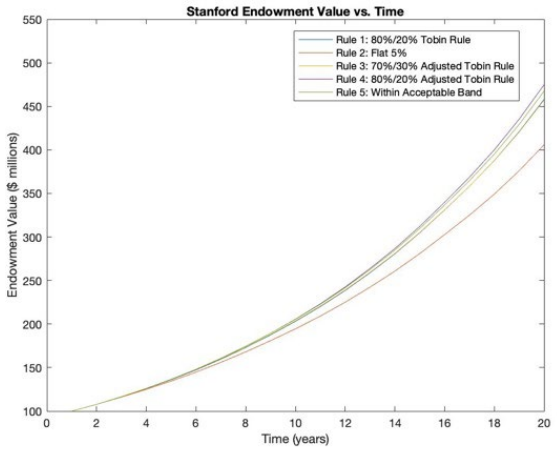
Panel B: Spending Percentage



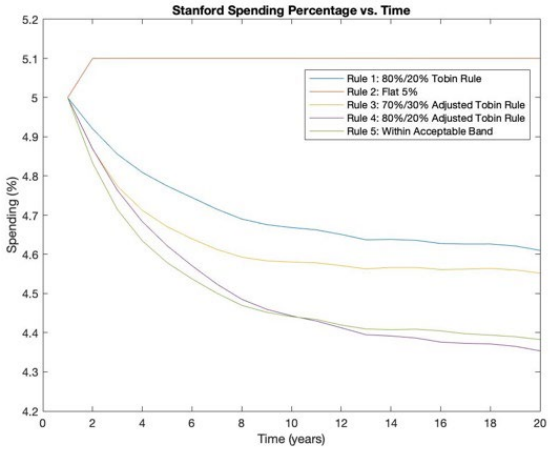
Panel C: Breakeven Return

Exhibit 4
Simulated value, spending, and breakeven return for the endowment of Stanford University

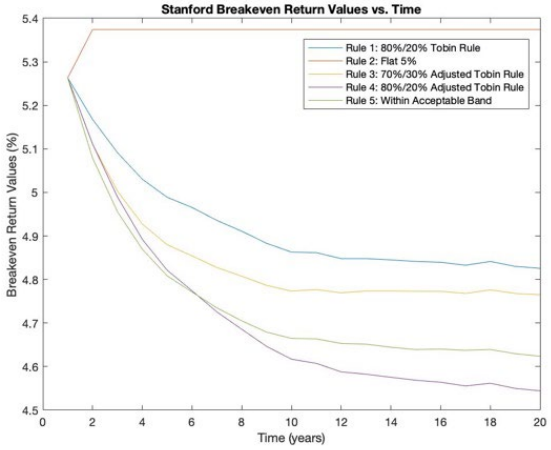
This figure shows simulated endowment value, spending percentage, and breakeven return for the endowment of Stanford University under five major spending rules. Rule 1 is the 80/20 Tobin rule, Rule 2 is the flat 5% rule adjusted for inflation, Rule 3 is the 70/30 adjusted Tobin rule, Rule 4 is the 80/20 adjusted Tobin rule, and Rule 5 is the within acceptable band rule.



Panel A: Endowment Value



Panel B: Spending Percentage

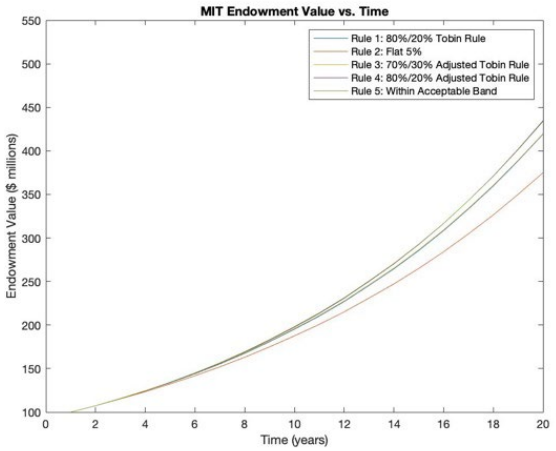


Panel C: Breakeven Return

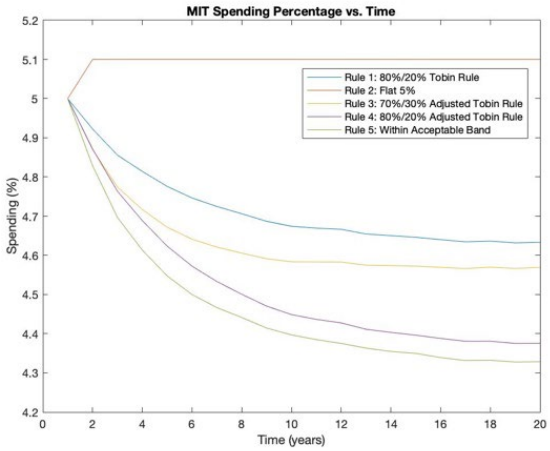
Exhibit 5

Simulated value, spending, and breakeven return for the endowment of Massachusetts Institute of Technology

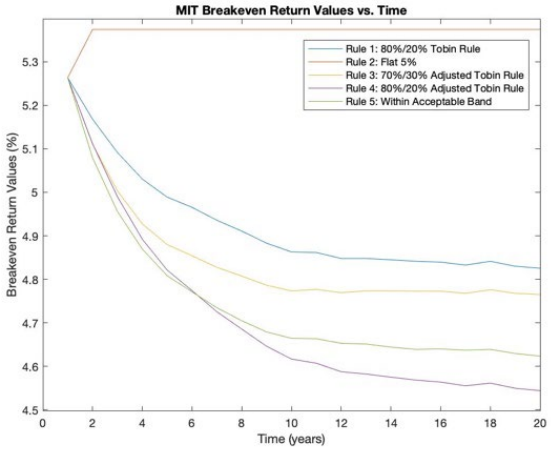
This figure shows simulated endowment value, spending percentage, and breakeven return for the endowment of MIT under five major spending rules. Rule 1 is the 80/20 Tobin rule, Rule 2 is the flat 5% rule adjusted for inflation, Rule 3 is the 70/30 adjusted Tobin rule, Rule 4 is the 80/20 adjusted Tobin rule, and Rule 5 is the within acceptable band rule.



Panel A: Endowment Value



Panel B: Spending Percentage



Panel C: Breakeven Return

Exhibit 6
Endowment performance for five major spending rules

This table reports performance metrics (relative change in value, average change in value, and benchmark spending) for the endowments of Harvard University, Yale University, Stanford University, and Massachusetts Institute of Technology (MIT). Performance metrics are computed for five spending rules, and are based on simulated returns taking into the account asset allocation data reported in Exhibit 1. Spending Rule 1 (SR1) is the 80/20 Tobin rule, SR2 is the flat 5% rule adjusted for inflation, SR3 is the 70/30 adjusted Tobin rule, SR4 is the 80/20 adjusted Tobin rule, and SR5 is the within acceptable band rule. Relative change in value is defined as the percentage change in spending amount over the percentage change in endowment value. Benchmark spending is defined as the percentage deviation from the 5% spending target.

Panel A: Relative Change in Value

	Spending Rules				
Endowment	SR1	SR2	SR3	SR4	SR5
Harvard University	-2.5	1.4	-3.6	-6.4	-8.8
Yale University	1.1	1.3	-0.6	-2.5	-2.9
Stanford University	0.7	1.3	-1.0	-3.0	-4.0
Mass Institute of Technology	-0.5	1.4	-2.2	-4.5	-6.3

Panel B: Average Change in Value

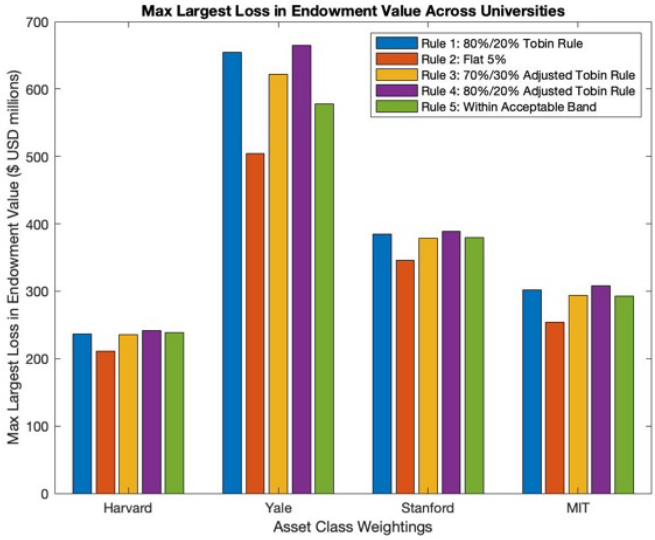
	Spending Rules				
Endowment	SR1	SR2	SR3	SR4	SR5
Harvard University	7.70%	7.20%	7.70%	7.90%	8.00%
Yale University	8.60%	8.10%	8.60%	8.80%	8.70%
Stanford University	8.10%	7.70%	8.20%	8.30%	8.30%
Mass Institute of Technology	7.60%	7.20%	7.70%	7.80%	7.90%

Panel C: Benchmark Spending

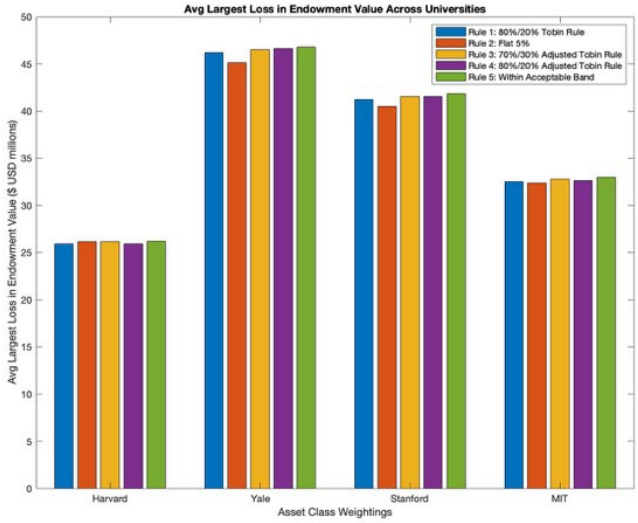
	Spending Rules				
Endowment	SR1	SR2	SR3	SR4	SR5
Harvard University	-6.40%	1.90%	-7.60%	-10.20%	-12.00%
Yale University	-6.10%	1.90%	-7.50%	-9.90%	-9.30%
Stanford University	-5.70%	1.90%	-7.20%	-9.60%	-9.70%
Mass Institute of Technology	-5.50%	1.90%	-7.10%	-9.30%	-10.40%

Exhibit 7
Analysis of largest losses under five major spending polices

This figure shows largest projected losses of Harvard, Yale, Stanford, and MIT's endowments under the five spending rules. Panel A shows the maximum largest loss, defined as the largest realized loss in simulations across all simulated paths. Panel B shows the average annual largest loss, defined as the average largest annual loss across simulation paths.



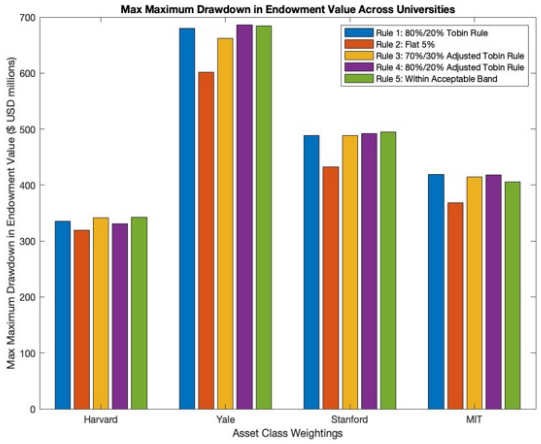
Panel A: Maximum Largest Loss



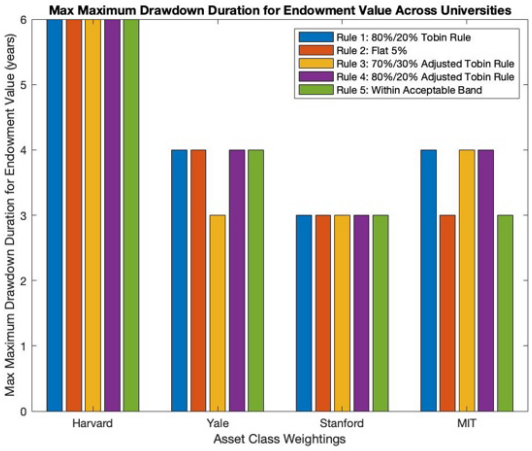
Panel B: Average Largest Loss

Exhibit 8
Analysis of drawdown under five major spending polices

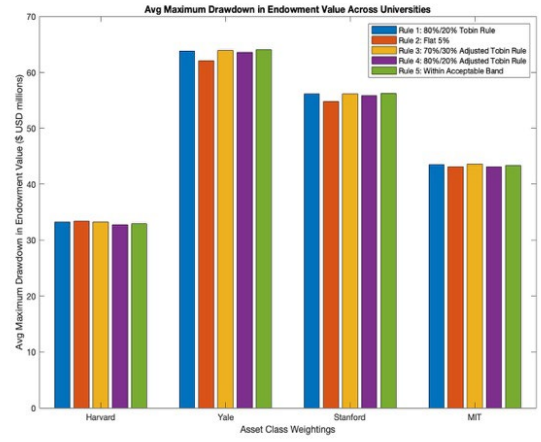
This figure shows maximum drawdown and its duration of Harvard, Yale, Stanford, and MIT's endowments under the five spending rules. Panel A shows the maximum largest drawdown, defined as the largest realized drawdown in simulations across all simulated paths. Panel B shows the duration of the largest realized drawdown shown in Panel B. Panel C shows the average annual largest drawdown, defined as the average largest drawdown across simulation paths. Panel D shows the duration of the average annual largest drawdown shown in Panel B.



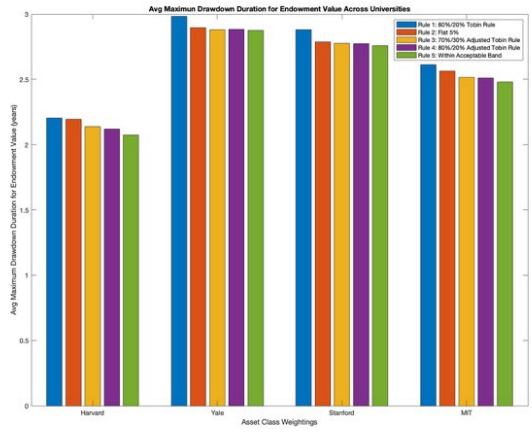
Panel A: Maximum Largest Drawdown



Panel B: Maximum Largest Drawdown Duration



Panel C: Average Largest Drawdown



Panel D: Average Largest Drawdown Duration

Exhibit 9
Comparison of asset allocations on major endowments with optimized portfolios

This table, in addition to the asset allocation data of Harvard, Yale, Stanford, and MIT from Exhibit 1, reports asset allocation for three optimized portfolios: the maximum Sharpe ratio portfolio, the minimum variance portfolio, and a mean-variance efficient portfolio for an investor with the risk aversion parameter of $\lambda = 2$.

Asset Class	Harvard FY 2019	Yale FY 2020	Stanford FY 2019	MIT FY 2019	Max Sharpe Ratio Portfolio	Min Variance Portfolio	Mean-Variance Efficient Portfolio
Domestic Equity	26.00%	2.75%	7.00%	11.00%	0.40%	0.10%	52.00%
International Equity	—	13.75%	20.00%	22.00%	—	—	—
Emerging Markets (EM) Equity	—	—	—	—	—	—	—
Fixed Income and Cash Instruments	7.00%	7.00%	8.00%	16.00%	53.30%	54.10%	—
Hedge Funds	33.00%	23.00%	20.00%	13.00%	40.90%	40.70%	11.90%
Private Equity	20.00%	38.00%	30.00%	23.00%	—	—	7.40%
Real Estate	8.00%	10.00%	8.00%	15.00%	5.30%	5.20%	13.90%
Real Assets	6.00%	5.50%	7.00%	1.00%	0.10%	—	14.80%

Exhibit 10
Root mean squared error

This figure shows the root mean squared error (RMSE) for the endowments of Harvard, Yale, Stanford, and MIT relative to the three portfolios constructed in Exhibit 9: the maximum Sharpe ratio portfolio, the minimum variance portfolio, and the mean-variance efficient portfolio for an investor with the risk aversion parameter of $\lambda = 2$.

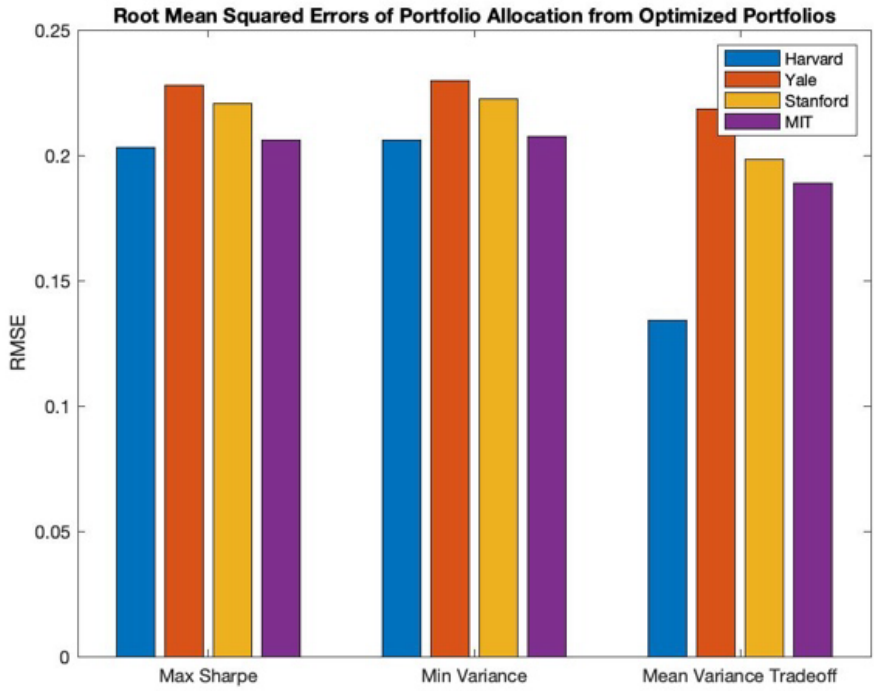


Exhibit 11
Frontiers of risk aversion and marginal utility of wealth

This figure shows frontiers of absolute risk aversion and the marginal utility of wealth per unit increase in spending for the endowments of Harvard, Yale, Stanford, and MIT. To construct these frontiers, we use Merton's (1993) model framework and asset allocation data for each endowment shown in Exhibit 1. We perturb a range of absolute risk aversion values, and for each value regress the marginal utility parameter to approximate the intertemporal preference of the endowment towards marginal wealth and spending.